USN

Second Semester B.E. Degree Examination, Dec.2017/Jan.2018
Engineering Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least two from each part.***

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) An equation of the form $y = Px + f(P)$ is known as,

A) CLARAUT's equation B) LAGRANGE's equation
C) CAUCHY's equation D) None of these

ii) If the given equation is solvable for if then it is of the form,

A) $y = f(x, P)$ B) $x = f(y, P)$ C) $x = f\left(\frac{y}{P}\right)$ D) $x = f\left(\frac{P}{y}\right)$

iii) If $L\frac{dI}{dt} + RI = E$ then $I =$ __________

A) $ER + Ce^{-Rt/L}$ B) $\frac{E}{R} + Ce^{-Rt/L}$ C) $ER + Ce^{L/Rt}$ D) $\frac{E}{R} + Ce^{-L/Rt}$

iv) The Clairaut's equation of $P = \log(Px - y)$ = _______

A) $y = Px + e^{P}$ B) $y = Px - e^{-P}$ C) $y = Px + e^{-P}$ D) $y = Px - e^{P}$

b. Solve $y = x + a\tan^{-1} P$. **(05 Marks)**

c. Obtain general solution of $Px^2 + P^2xy - xy - Py^2 = 2P$. **(06 Marks)**

d. Solve $y = x\left[P + \sqrt{1 + P^2}\right]$. **(05 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) $e^{-x}\left(C_1 \cos\sqrt{2}x + C_2 \sin\sqrt{2}x\right) + C_3 e^{3x}$ is the general solution of,

A) $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 9y = 0$ B) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 9y = 0$

C) $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 9y = 0$ D) None of these

ii) The particular integral of $(D + a)^2 y = e^{-ax}$ is,

A) $e^{-ax}\frac{x}{2}$ B) $e^{-ax}\frac{x^2}{2}$ C) $e^{ax}\frac{x^2}{2}$ D) $e^{ax}\frac{x}{2}$

iii) The complimentary function of the differential equation, $D^2(D-1)^2 y = e^x$ is,

A) $C_1 + C_2 + C_3e^x + C_4e^x$ B) $C_1x + C_2 + C_3x + C_4$
C) $C_1x + C_2 + (C_3x + C_4)e^x$ D) $(C_1x + C_2)e^{-x} + (C_3x + C_4)e^x$

iv) If $f(D) = D^2 + 3$, then $\frac{1}{f(D)}\sin 2x$ is,

A) $7\sin 2x$ B) $-\cos 2x$ C) $\sin 2x$ D) $-\sin 2x$

b. Solve $\frac{d^2y}{dx^2} + a^2y = \sin(ax + b)$. **(05 Marks)**

c. Solve $(D^3 + 2D^2 + D)y = x^2e^{2x} + \cos^2 x$. **(06 Marks)**

d. Solve $Dx - (D + 1)y = -e^t$.
$x + (D - 1)y = e^{2t}$ **(05 Marks)**

3 a. Choose the correct answers for the following : (04 Marks)

i) The differential equation of the form, $\left(x^n D^n + a_1 x^{n-1} D^{n-1} + ... + a_{n-1} xD + a_n\right)y = F(x)$ and $a_i's$, I = 1, 2, 3, ...n are constants, and F(x) is a function of x is known as.

A) Legendrey's Linear equation
B) Cauchy's linear equation
C) Bessel's equation
D) Reducible to Bessel's equation

ii) The Wronskian of cos2x and sin2x is,

A) 0 B) 1 C) 2 D) −2

iii) In a differential equation of the form, $P_0(x)\frac{d^2y}{dx^2} + P_1(x)\frac{dy}{dx} + P_2(x)y = 0$, If $P_0(a) \neq 0$, then x = a is called an,

A) Ordinary point B) Singular point C) Both (A) and (B) D) None of these

iv) To transform the equation, $(ax+b)^2\frac{d^2y}{dx^2} + (ax+b)\frac{dy}{dx} + y = e^x$ into linear differential equation with constant coefficient, the substitution is,

A) $ax + b = e^{at+b}$ B) $ax + b = e^{at}$ C) $ax + b = e^t$ D) $ax + b = e^{-t}$

b. Solve by variation of parameters, $\frac{d^2y}{dx^2} + a^2y = \sec ax$. (05 Marks)

c. Solve the differential equation, $(2x+3)^2\frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x$. (05 Marks)

d. Solve by Frobenius method, $xy'' + y' + xy = 0$ (06 Marks)

4 a. Choose the correct answers for the following : (04 Marks)

i) If the number of arbitrary constants is more than the number of independent variables then the partial differential equation is of,

A) Only first order
B) Only of second order
C) Second or Higher orders
D) None of these

ii) The partial differential equation obtained by eliminating a and b from $z = ax^2 + by^2$ i.

A) $z = px + qy$ B) $2z = px + qy$ C) $z = px - qy$ D) $z = px + qy +$

iii) The P.D.E obtained by elimination of f and g from $z = f(x)g(x)$ is,

A) $pq = zr$ B) $pq = zs$ C) $pq = rs$ D) $pqr = z$

iv) A linear partial differential equation of the first order of the form, $Pp + Qq = R$, where P, Q, R are functions of x, y, z is known as,

A) Lagrange's linear first order P.D.E
B) Lagrange's linear second order P.D.E
C) Lagrange's ordinary linear differential equation
D) Clairant's linear differential equation

b. Form partial differential equation by eliminating F, $F(x+y+z, x^2+y^2+z^2) = 0$ (06 Marks)

c. Solve $x(y^2 - z^2)p + y(z^2 - x^2)q - z(x^2 - y^2) = 0$. (05 Marks)

d. Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$, given when x = 0, $\frac{\partial z}{\partial x} = a\sin y$, $\frac{\partial z}{\partial y} = 0$. (05 Marks)

PART – B

5 a. Choose the correct answers for the following : (04 Marks)

i) The integral $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-y^2}} f(x,y)dxdy$ after changing order of integration becomes,

A) $\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} f(x,y)dxdy$ B) $\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} f(x,y)dydx$ C) $\int_{0}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} f(x,y)dydx$ D) None of these

ii) The value of $\int_{3}^{4}\int_{1}^{2}\frac{dydx}{(x+y)^2}=$

A) $\log\frac{25}{24}$ B) $\log\frac{24}{25}$ C) 0 D) 1

iii) The relation between β and τ functions is,

A) $\tau(m,n)=\frac{\beta(m)\cdot\beta(n)}{\beta(m+n)}$ B) $\beta(m,n)=\frac{\tau(m)\cdot\tau(n)}{\tau(m+n)}$

C) $\beta(m,n)=\frac{\tau(m)\tau(n)}{\tau(m-n)}$ D) $\beta(m,n)=\frac{\tau(m)}{\tau(n)}$

iv) The value of $\tau\left(\frac{1}{2}\right)=$ ______

A) 1.772 B) 2.772 C) 1 D) 0

b. Change order of integration, $I=\int_{0}^{1}\int_{x}^{\sqrt{2-x^2}}\frac{x}{\sqrt{x^2+y^2}}dydx$ and hence evaluate it. **(06 Marks)**

c. Evaluate $\int_{1}^{e}\int_{1}^{\log y}\int_{1}^{e^x}\log z\,dz\,dx\,dy$. **(05 Marks)**

d. Prove that $\beta\left(m,\frac{1}{2}\right)=2^{2m-1}\beta(m,n)$. **(05 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\int_{C}y^2dx-2x^2dy$ along the parabola $y=x^2$ from $(0,0)$ to (2, 4) is,

A) $\frac{5}{48}$ B) $-\frac{5}{48}$ C) $-\frac{48}{5}$ D) $\frac{5}{24}$

ii) If $\phi(x,y)$, $\psi(x,y)$, ϕ_y, ψ_x are continuous in a region E of the xy plane bounded by a closed curve C, then

A) $\int_{C}\phi dx+\psi dy=\iint_{E}\left(\frac{\partial\psi}{\partial x}-\frac{\partial\phi}{\partial y}\right)dxdy$ B) $\int_{C}\phi dx+\psi dy=\iint_{E}\left(\frac{\partial\upsilon}{\partial x}-\frac{\partial\phi}{\partial y}\right)dxdy$

C) $\int_{C}\phi dx+\psi dy=\iint_{E}\left(\frac{\partial\phi}{\partial x}+\frac{\partial\phi}{\partial y}\right)dxdy$ D) $\int_{C}\phi dx+\psi dy=\iint_{E}\left(\frac{\partial o}{\partial x}+\frac{\partial\psi}{\partial y}\right)dxdy$

iii) If S be an open surface bounded by a closed curve C and $F=f_1\hat{i}+f_2\hat{j}+f_3\hat{k}$, a differential vector function and $N=\cos\alpha\hat{i}+\cos\beta\hat{j}+\cos\gamma\hat{k}$ is a unit external normal then $\int_{C}F.dR=$ ______

A) $\int_{S}\text{grad}F.Nds$ B) $\int_{S}\text{div}F.Nds$ C) $\int_{S}\text{grad}F\times Nds$ D) $\int_{S}\text{curl}F.Nds$

iv) If F is a continuous differential vector function in the region E bounded by the closed surface S, then $\int_{C}F\cdot NdS=\int_{E}\text{div}FdV$. This statement is,

A) Green's statement B) Gauss divergence theorem

C) Stoke's theorem D) None of these

b. Using Green's theorem evaluate $\int_{C}(y-\sin x)dx+\cos xdy$ where C is plane triangle enclosed by the lines y = 0, $x=\frac{\pi}{2}$ and $y=\frac{2}{\pi}x$. **(05 Marks)**

c. Using stokes theorem evaluate $\int_C (x+y)dx + (2x-z)dy + (y+z)dz$, where C the boundary of the triangle with verticies (2, 0, 0), (0, 3, 0), (0, 0, 6). (05 Marks)

d. Verify divergence theorem for, $F = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ over the rectangular parallelepiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$. (06 Marks)

7 a. Choose the correct answers for the following : (04 Marks)

i) If $L\{f(t)\} = F(s)$ then $L\left\{\frac{d^2}{dt^2} f(t)\right\} =$ ______

A) $S^2F(s) + sf(0) + f'(0)$ B) $S^2F(s) - sf(0) - f'(0)$ C) $S^2F(s) - f(0)$ D) $SF(s) - f(0)$

ii) $L\{t\cos at\} =$ ________

A) $\frac{s^2+a^2}{(s^2-a^2)^2}$ B) $\frac{s^2}{(s^2-a^2)^2}$ C) $\frac{s^2-a^2}{s^2+a^2}$ D) $\frac{s^2-a^2}{(s^2+a^2)^2}$

iii) If $f(t+T) = f(t)$, then $L\{f(t)\} =$ _____

A) $\frac{1}{1-e^{-st}}\int_0^T e^{-st} f(t)dt$ B) $\frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)dt$ C) $\frac{1}{1-e^{-sT}}\int_0^T e^{-sT} f(T)dt$ D) None of these

iv) The unit step function, $u(t-a) =$ ________

A) $\begin{cases} 0, & t > a \\ 1, & t < a \end{cases}$ B) $\begin{cases} -1, & t < a \\ 1, & t \ge a \end{cases}$ C) $\begin{cases} 0, & t < a \\ 1, & t \ge a \end{cases}$ D) $\begin{cases} 0, & t = a \\ 1, & t < a \,\&\, t > a \end{cases}$

b. Evaluate $L\left\{\int_0^t \frac{e^{-t}\sin t}{t} dt\right\}$. (06 Marks)

c. Find the Laplace transform of the periodic function, $f(t) = \begin{cases} 1, & 0 \le t \le 2 \\ -1, & 2 \le t < 4 \end{cases}$. (05 Marks)

d. Express f(t) in terms of unit step function & find $L\{f(t)\}$, where $f(t) = \begin{cases} 0, & 0 < t < 1 \\ t-1, & 1 < t < 2 \\ 1, & t > 2 \end{cases}$. (05 Marks)

8 a. Choose the correct answers for the following : (04 Marks)

i) If $L^{-1}\{F(s)\} = f(t)$, then $L^{-1}\{F(s-a)\} =$ ________

A) $e^{at}f(t)$ B) $e^t f(at)$ C) $e^{at}f(at)$ D) $e^{-at}f(t)$

ii) If $L^{-1}\{F(s)\} = f(t)$, $L^{-1}\{G(s)\} = g(t)$, then $L^{-1}\{F(s)G(s)\} =$ ________,

A) $\int_0^t f(u)g(u)du$ B) $\int_0^t f(t-u)g(t-u)du$ C) $\int_0^t f(u)g(t-u)du$ D) None of these

iii) $L^{-1}\left\{\frac{s^2-3s+4}{s^3}\right\} =$ ________

A) $\frac{t^2}{2} - 3t + 1$ B) $2t^2 - 3t + 1$ C) $2t^2 - 2t + 1$ D) $2(t^2 - t + 1)$

iv) $L^{-1}\left\{\frac{1}{(s-a)^2 + b^2}\right\} =$ ________

A) $e^{at}\sin bt$ B) $\frac{1}{a}e^{at}\sin bt$ C) $\frac{e^{at}}{b}\sin bt$ D) $\frac{e^{at}}{a^2}\sin bt$

b. Find $L^{-1}\left\{\log\frac{s^2+1}{s(s+1)}\right\}$. (05 Marks)

c. Using convolution theorem, evaluate $L^{-1}\left\{\frac{1}{s^3(s^2+1)}\right\}$. (05 Marks)

d. Solve using L.T $y''' + 2y'' - y' - 2y = 0$. Given $y(0) = y'(0) = 0$ and $y''(0) = 6$. (06 Marks)